MATH 376-02 Advanced Linear Algebra

Homework

Below, F is a field and V is a vector space over F.

1. **Due 1/23** If $a \in F$, then $-a$ is unique and a^{-1} is unique if $a \neq 0$.
2. **Due 1/23** For all $a \in F$, $a \cdot 0 = 0$.
3. **Due 1/23** The additive identity 0 cannot have a multiplicative inverse in F.
4. **Due 1/25** F^∞ is a subspace of $F^{\infty\infty}$.
5. **Due 1/25** If E is an extension field of F, then E is a vector space over F.
6. **Due 1/30** If $0 \in S \subseteq V$, then S is linearly dependent.
7. **Due 1/30** If $S \subseteq T \subseteq V$ and S is linearly dependent, then T is linearly dependent.
8. **Due 1/30** A linear transformation $T : V \to W$ is one-to-one if and only if $\ker T = \{0\}$.
9. **Due 1/30** If $T : F^n \to F^m$ is a linear transformation and $B = [T(e_1)T(e_2)\ldots T(e_n)]$, then $T = T_B$.
10. **Due 1/30** An $n \times n$ matrix A is invertible if and only if T_A is invertible. [Hint: the previous exercise may come in handy!]
11. **Due 2/13** (Lemma 1.4.4) If $W_1, \ldots, W_k$ are subspaces of the vector space V, then they are independent if and only if $W_i \cap \left(\sum_{j \neq i} W_j \right) = \{0\}$ for each $i, 1 \leq i \leq k$.
12. **Due 2/13** (Lemma 1.4.5) Let V be a vector space, and $W_1, \ldots, W_k$ subspaces of V. Then $V = W_1 \bigoplus \cdots \bigoplus W_k$ if and only if for all $v \in V$ there exist unique $w_1 \in W_1, \ldots, w_k \in W_k$ such that $v = w_1 + \ldots + w_k$.
13. **Due 2/17** (Lemma 1.5.2) Let X be an affine subspace of V parallel to the subspace U. Then for all $x \in X$, $U = \{x' - x : x' \in X\}$.
14. **Due 2/17** (Remark 1.5.3) An affine subspace X of V is a subspace of V if and only if $0 \in X$.
15. **Due 2/17** (Proposition 1.5.4) If $X \subseteq V$ and U is a subspace of V, then X is an affine subspace of V parallel to U if and only if there exists $x \in U$ such that $X = x + U$ (where $x + U = \{x + u | u \in U\}$).
16. **Due 2/17** (See Definition 1.5.12) The canonical projection is a linear transformation. (Note that you will need to show that π is well defined, too.)
17. **Due 2/20** If $\pi : V \to V/W$ is the canonical projection, then $\ker \pi = W$.

18. **Due 3/3** If U^* is a subspace of V^*, then the annihilator of U^* is a subspace of V.

19. **Due 3/3** A linear transformation $T : V \to W$ is surjective if and only if its cokernel is trivial.

20. **Due 3/8** If V is a vector space and U_1 and U_2 are subspaces of V with $U_1 \subseteq U_2$, then $\mathrm{Ann}^*(U_2) \subseteq \mathrm{Ann}^*(U_1)$.

21. **Due 3/8** (Theorem 1.6.15 (1)) Let U be a subspace of V. Then $\mathrm{Ann}(\mathrm{Ann}^*(U)) = U$.

22. **Due 3/8** Let $T : V \to W$ be a linear transformation. Then $\mathrm{Ann}^*(\mathrm{Im}\ T) = \ker T^*$.

23. **Due 3/8** Let $T : V \to W$ be a linear transformation. Then $\mathrm{Im}\ T = \mathrm{Ann}(\ker T^*)$.

24. **Due 4/7** (Corollary 3.3.9 (1)) Let A be an $n \times n$ matrix over F. Then $(\mathrm{Adj}(A))A = A(\mathrm{Adj}(A)) = \det(A)I$. (Note typo in book.)

25. **Due 4/7** (Corollary 3.3.9 (2)) Let A be an $n \times n$ matrix over F. If A is invertible, then $A^{-1} = \dfrac{1}{\det(A)}\mathrm{Adj}(A)$.

26. **Due 4/7** Let V be a finite dimensional vector space and $T : V \to V$ a linear transformation. Then $\det(T)$ is well defined (i.e., independent of the choice of basis). (See Definition 3.3.15.)

27. **Due 4/12** (Example 4.1.5(2)) Show that if $V =^r F^{\infty\infty}$ and $L : V \to V$ is left-shift, then for every $\lambda \in F$, E^k_λ is k-dimensional.

28. **Due 4/26** Let X be a set, and let $T = \{A : A \subseteq X\}$. Prove that T is a topology on X. (This is the **discrete** topology.)

29. **Due 4/26** Prove that $T = \{B_r(x) : r > 0, x \in \mathbb{R}^n\}$ is a basis for a topology on $\mathbb{R}^n$.

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